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## MATHEMATICAL STUDIES <br> STANDARD LEVEL <br> PAPER 2

Thursday 6 May 2010 (morning)
1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- A graphic display calculator is required for this paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 15]
(a) Phoebe chooses a biscuit from a blue tin on a shelf. The tin contains one chocolate biscuit and four plain biscuits. She eats the biscuit and chooses another one from the tin. The tree diagram below represents the situation with the four possible outcomes where $A$ stands for chocolate biscuit and $B$ for plain biscuit.

(i) Write down the value of $a$.
(ii) Write down the value of $b$.
(iii) Find the probability that both biscuits are plain.

## (Question 1 continued)

On another shelf there are two tins, one red and one green. The red tin contains three chocolate biscuits and seven plain biscuits and the green tin contains one chocolate biscuit and four plain biscuits. Andrew randomly chooses either the red or the green tin and randomly selects a biscuit.
(b) Copy and complete the tree diagram below.

(c) Find the probability that
(i) he chooses a chocolate biscuit;
(ii) he chooses a biscuit from the red tin given that it is a chocolate biscuit.
[6 marks]
2. [Maximum mark: 17]

Alex and Kris are riding their bicycles together along a bicycle trail and note the following distance markers at the given times.

| Time $(t$ hours $)$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance $(d \mathrm{~km})$ | 57 | 65 | 72 | 81 | 89 | 97 | 107 |

(a) Draw a scatter diagram of the data. Use 1 cm to represent 1 hour and 1 cm to represent 10 km .
(b) Write down for this set of data
(i) the mean time, $\bar{t}$;
(ii) the mean distance, $\bar{d}$.
(c) Mark and label the point $\mathrm{M}(\bar{t}, \bar{d})$ on your scatter diagram.
(d) Draw the line of best fit on your scatter diagram.
(e) Using your graph, estimate the time when Alex and Kris pass the 85 km distance marker. Give your answer correct to one decimal place.
(f) Write down the equation of the regression line for the data given.
(g) (i) Using your equation calculate the distance marker passed by the cyclists at 10.3 hours.
(ii) Is this estimate of the distance reliable? Give a reason for your answer.
3. [Maximum mark: 20]

## Give all answers in this question to the nearest whole currency unit.

Ying and Ruby each have 5000 USD to invest.
Ying invests his 5000 USD in a bank account that pays a nominal annual interest rate of 4.2 \% compounded yearly. Ruby invests her 5000 USD in an account that offers a fixed interest of 230 USD each year.
(a) Find the amount of money that Ruby will have in the bank after 3 years.
(b) Show that Ying will have 7545 USD in the bank at the end of 10 years.
(c) Find the number of complete years it will take for Ying's investment to first exceed 6500 USD.
(d) Find the number of complete years it will take for Ying's investment to exceed Ruby's investment.

Ruby moves from the USA to Italy. She transfers 6610 USD into an Italian bank which has an exchange rate of 1 USD $=0.735$ Euros. The bank charges $1.8 \%$ commission.
(e) Calculate the amount of money Ruby will invest in the Italian bank after commission.

Ruby returns to the USA for a short holiday. She converts 800 Euros at a bank in Chicago and receives 1006.20 USD. The bank advertises an exchange rate of 1 Euro $=1.29$ USD.
(f) Calculate the percentage commission Ruby is charged by the bank.
4. [Maximum mark: 24]
(a) A gardener has to pave a rectangular area 15.4 metres long and 5.5 metres wide using rectangular bricks. The bricks are 22 cm long and 11 cm wide.
(i) Calculate the total area to be paved. Give your answer in $\mathrm{cm}^{2}$.
(ii) Write down the area of each brick.
(iii) Find how many bricks are required to pave the total area.
(b) The gardener decides to have a triangular lawn ABC , instead of paving, in the middle of the rectangular area, as shown in the diagram below.


The distance AB is 4 metres, AC is 6 metres and angle BAC is $40^{\circ}$.
(i) Find the length of BC.
(ii) Hence write down the perimeter of the triangular lawn.
(iii) Calculate the area of the lawn.
(iv) Find the percentage of the rectangular area which is to be lawn.

## (Question 4 continued)

(c) In another garden, twelve of the same rectangular bricks are to be used to make an edge around a small garden bed as shown in the diagrams below. FH is the length of a brick and C is the centre of the garden bed. M and N are the midpoints of the long edges of the bricks on opposite sides of the garden bed.

diagram not to scale
(i) Find the angle FCH.
(ii) Calculate the distance MN from one side of the garden bed to the other, passing through C.

The garden bed has an area of $5419 \mathrm{~cm}^{2}$. It is covered with soil to a depth of 2.5 cm .
(d) Find the volume of soil used.

It is estimated that 1 kilogram of soil occupies $514 \mathrm{~cm}^{3}$.
(e) Find the number of kilograms of soil required for this garden bed.
5. [Maximum mark: 14]

A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, $h$ represents the height of the can in cm and $x$, the radius of the base of the can in cm .

diagram not to scale

The volume of the dog food cans is $600 \mathrm{~cm}^{3}$.
(a) Show that $h=\frac{600}{\pi x^{2}}$.
[2 marks]
(b) (i) Find an expression for the curved surface area of the can, in terms of $x$. Simplify your answer.
(ii) Hence write down an expression for $A$, the total surface area of the can, in terms of $x$.
[4 marks]
(c) Differentiate $A$ in terms of $x$.
(d) Find the value of $x$ that makes $A$ a minimum.
(e) Calculate the minimum total surface area of the dog food can.

